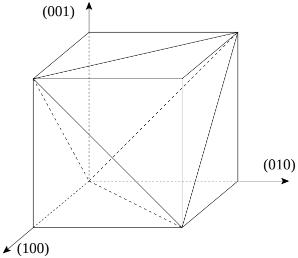
Slip systems in a FCC single crystal (1/2)

A slip system s is characterised by its plan (vector $\underline{\mathbf{n}}$) and the slip direction (vector $\underline{\mathbf{m}}$). The resolved shear stress τ^s is computed as:

$$\tau^s = \sigma_{ij} n_i m_j = \frac{1}{2}\sigma_{ij}(n_i m_j + m_i n_j) = \sigma_{ij} m_{ij}$$

with the *orientation tensor* defined by $\underset{\sim}{\mathbf{m}} = \frac{1}{2}(\underline{\mathbf{n}} \otimes \underline{\mathbf{m}} + \underline{\mathbf{m}} \otimes \underline{\mathbf{n}})$



The four octhedral planes of a cubic crystal

Schmid's law assumes that slip starts on a given slip system when the resolved shear stress reaches a threshold, called *critical resolved shear stress*, τ_c. The model is then made of a collection of linear stress dependent criteria:

$$|\tau^s| - \tau_c = 0 \quad \text{or} \quad \underset{\sim}{\sigma} : \underset{\sim}{\mathbf{m}}^s - \tau_c = 0$$

Slip systems in a FCC single crystal (2/2)

Each of the four octahedral planes contains three systems, defined as follows: (planes denoted by $\underline{\mathbf{n}}$, directions denoted by $\underline{\mathbf{m}}$):

num syst	1	2	3	4	5	6	7	8	9	10	11	12
$\sqrt{3}n_1$	1	1	1	1	1	1	-1	-1	-1	1	1	1
$\sqrt{3}n_2$	1	1	1	-1	-1	-1	1	1	1	1	1	1
$\sqrt{3}n_3$	1	1	1	1	1	1	1	1	1	-1	-1	-1
$\sqrt{2}m_1$	-1	0	-1	-1	0	1	0	1	1	-1	1	0
$\sqrt{2}m_2$	0	-1	1	0	1	1	-1	1	0	1	0	1
$\sqrt{2}m_3$	1	1	0	1	1	0	1	0	1	0	1	1

Orientation tensors of a FCC tensor

num syst	1	2	3	4	5	6	7	8	9	10	11	12
$\sqrt{6}m_{11}$	-1	0	-1	-1	0	1	0	-1	-1	-1	1	0
$\sqrt{6}m_{22}$	0	-1	1	0	-1	-1	-1	1	0	1	0	1
$\sqrt{6}m_{33}$	1	1	0	1	1	0	1	0	1	0	-1	-1
$2\sqrt{3}m_{12}$	-1	-1	0	1	1	0	1	0	1	0	1	1
$2\sqrt{3}m_{23}$	1	0	1	-1	0	1	0	1	1	-1	1	0
$2\sqrt{3}m_{31}$	0	1	-1	0	1	1	-1	1	0	1	0	1

- Assuming that the only non zero terms in the stress tensor are σ_{11}, σ_{12} and σ_{21}, the criterion writes:

$$|\sigma_{11} m_{11} + 2\sigma_{12} m_{12}| - \tau_c = 0$$

- Assuming that the only non zero terms in the stress tensor are σ_{11} and σ_{33}, the criterion writes:

$$|\sigma_{11} m_{11} + \sigma_{33} m_{33}| - \tau_c = 0$$

Resolved shear stress for σ_{11}–σ_{12} and σ_{11}–σ_{33} for a FCC single crystal

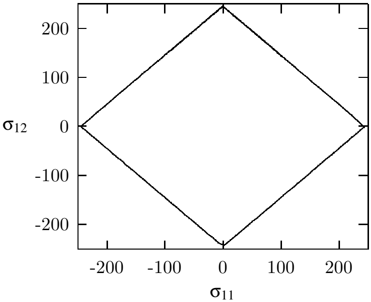
For the stresses σ_{11} et σ_{12}, the values of τ^s are respectively:

syst #	1	2	3	4	5	6
τ^s	$-\sigma_{11} - \sigma_{12}$	$-\sigma_{12}$	$-\sigma_{11}$	$-\sigma_{11} + \sigma_{12}$	σ_{12}	σ_{11}
syst #	7	8	9	10	11	12
τ^s	σ_{12}	$-\sigma_{11}$	$-\sigma_{11} + \sigma_{12}$	$-\sigma_{11}$	$\sigma_{11} + \sigma_{12}$	σ_{12}

And for the stresses σ_{11} et σ_{33},

syst #	1	2	3	4	5	6
τ^s	$-\sigma_{11} + \sigma_{33}$	σ_{33}	$-\sigma_{11}$	$-\sigma_{11} + \sigma_{33}$	σ_{33}	σ_{11}
syst #	7	8	9	10	11	12
τ^s	σ_{33}	$-\sigma_{11}$	$-\sigma_{11} + \sigma_{33}$	$-\sigma_{11}$	$\sigma_{11} - \sigma_{33}$	$-\sigma_{33}$

Initial yield locus of a FCC single crystal in the σ_{11}–σ_{12} plane



(Figure made with τ_c=100 MPa)

In tension–shear, the initial yield locus is defined by four systems

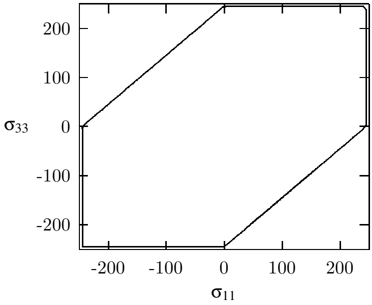
- *the systems 1 et 11 give:*

$$|\sigma_{11} + \sigma_{12}| = \tau_c\sqrt{6}$$

- *the systems 4 et 9 give:*

$$|\sigma_{11} - \sigma_{12}| = \tau_c\sqrt{6}$$

Initial yield locus of a FCC single crystal in the σ_{11}–σ_{33} plane



(Figure made with τ_c=100 MPa)

In biaxial tension, all the systems are involved

- *the systems 1, 4, 9 et 11 :*

$$|\sigma_{11} - \sigma_{33}| = \tau_c \sqrt{6}$$

- *the systems 3, 6, 8, 10 :*

$$|\sigma_{11}| = \tau_c \sqrt{6}$$

- *the systems 2, 5, 7, 13 :*

$$|\sigma_{33}| = \tau_c \sqrt{6}$$