Sheet folding and elastic springback effect

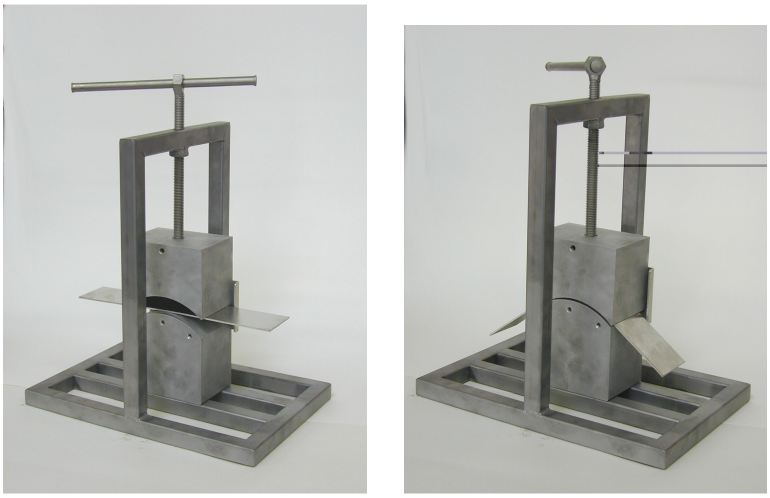

FIG. 1 – A simple device to apply circular bending

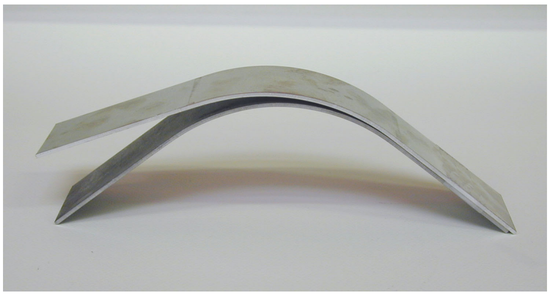

FIG. 2 – The final shape depends on material type (aluminium, steel) and on geometry

This mini-project treats of the sheet folding process, with very simple modelling conditions based on pure (circular) bending solutions. One objective is to make evidence of the importance of hardening laws, by using more realistic rules that include Bauschinger effects when predicting the final shape of the product after the elastic return produced by the unloading stage.

The project is organised in two steps :

- one analytical approach, as a standard exercise, considering the perfectly plastic case,
- extended conditions with various isotropic and kinematic hardening rules, with simple automatic simulations.

The study is based on circular bending conditions, as shown on figure 1, on sheets of 1 or 2 mm thickness, in aluminium or steel, with a circular preform allowing to obtain under load a curvature radius of 70 mm. Values could be changed but, under these conditions, the problem meets the small strain assumption.

Elastic limit is overpassed during the loading. When unloading, a residual stress field is established and the sheet does not recover its initial plane shape. However, it does not maintain its 70 mm curvature radius from loading condition, the final shape having a much larger radius. Final shapes are different for steel and aluminium. Figure 2 illustrates this fact. The objective of the project is to predict the applied bending moment and the final shape of the sheet.

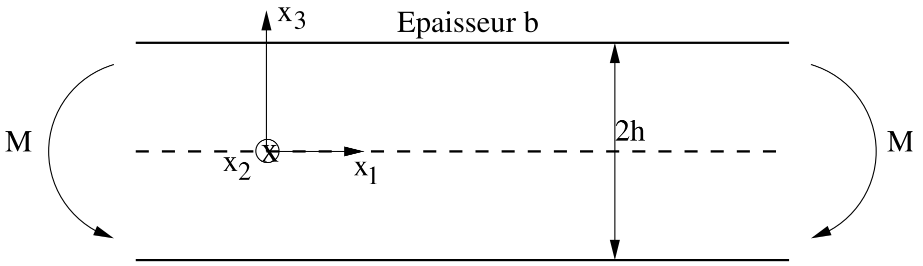

FIG. 3 – Coordinates, geometry and beam loading

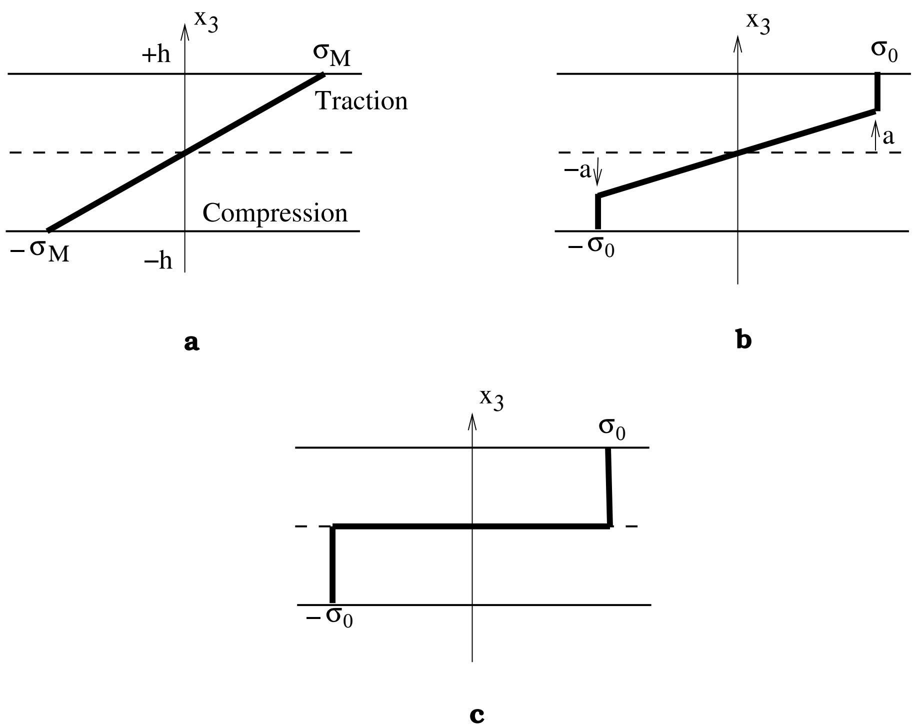

FIG. 4 – σ_{11} stress repartition in a beam in pure bending : (a) elasticity, (b) during plastification, (c) at limit load

0.1 Part I : Closed form solutions (beam theory)

The beam of figure 3 possess a rectangular section (x_2;x_3) of $2h$ height and b width. It is loaded in pure bending (circular bending), neglecting shear effects and we assume that any plane section of normal x_1 remains plane. The material behaviour is elastic (E, ν) and perfectly plastic (yield at σ_y).

1. What are the stress and strain distributions in elasticity ? Express the bending moment M in terms of geometrical quantities and bending inertia $I = 2bh^3/3$.
2. Obtain the moment M_e at the onset of plastic yielding. Figure 24 indicates schematically the stress distribution in a section when $M = M_e$, when $M_e < M < M_\infty$ and at the limit load when $M = M_\infty$ (limit load corresponds to the total plastification of the beam, without any possible increase of bending moment). Obtain the bending moment as a function of σ_y and geometric parameters, with $-a \leq x_3 \leq a$ the elastic region in the section.
3. What is the solution when unloading ($M = 0$),
   – i) in case where the maximum moment is $M_m < M_e$,
   – ii) when the maximum moment is such that $M_e < M_m < M_\infty$?

Show that a residual stress field does exist in this last case (after unloading). Indicate on a figure its repartition in the section for $M_e < M_m < M_\infty$ and when M_m was just at the limit load ($M_m = M_\infty$).

4. The neutral line, for which stress is zero valued, is at the middle section ($x_3 = 0$). We assume that the kinematics involves two fields, the vertical displacement $W(x_1)$ of points on neutral line and the rotation angle $\theta(x_1)$ of any section perpendicular to neutral line. The displacement of any point is like :

$$u_1(x_1, x_3) = \theta(x_1)x_3$$

$$u_3(x_1, x_3) = W(x_1)$$

As shear strain ε_{13} is zero valued, obtain the relation that gives the vertical displacement W from the rotation. Deduce the axial strain ε_{11}

Express the bending moment as a function of Young's modulus, bending inertia and the rotation in the elastic case. Under plastic flow, rotation is still imposed by the central elastic nucleus. Plane section remains plane and its orientation is given by the slope between $-a$ and a, in which zone :

$$\varepsilon_{11} = \frac{\sigma_{11}}{E} = \frac{\sigma_y}{E}\frac{x_3}{a}$$

$$\theta_{,1} = \frac{\sigma_y}{Ea}$$

Express the curvature $1/\rho = W_{,11}$ in elasticity and under plastic flow, where ρ is the radius of curvature of the sheet.

5. Express the bending moment during plastic flow (from question 2) and take into account the relation between a, the radius of curvature ρ and material characteristics.

   We assume $\sigma_y = 200$ MPa, $E = 72400$ MPa, $2h = 2$ mm (aluminium alloy). Check the curve of figure 0.1 for values of ρ indicated. Give values of M_e and corresponding radius of curvature, as well as the moment at limit load when collapse takes place. These quantities are expressed per unit width of the sheet ($b = 1$).

   We assume the radius of the preform to be 70 mm. Give the strain value at the top surface of the sheet for a sheet of 1 mm thickness and for a sheet of 2 mm thickness.

6. Considering that loading has practically attained the limit load, demonstrate the relation below that expresses the radius ρ_u after unloading in terms of the radius ρ_m under load (identical with the preform radius) :

$$\frac{1}{\rho_u} = \frac{1}{\rho_m} - \frac{3\sigma_y}{2hE}$$

0.2 Part II : Using the simulator and more realistic hardening rules

A simple simulation sheet is available following *this link*.
The Graphical User Interface allows to solve the above problem when taking into account various hardening laws. ρ_{max} is the minimum radius of curvature attained at the maximum of the loading (in fact the radius of the preform). The constitutive law is written in the rate independent plasticity framework, with a von Mises yield criterion and a combination of isotropic hardening R (linear or non linear) and non linear kinematic hardening X. Equations are written for uniaxial conditions on the simulation sheet, provided these conditions are present in the bending example. Please do not confuse material parameter b with the width of the sheet (also b) in Part I.

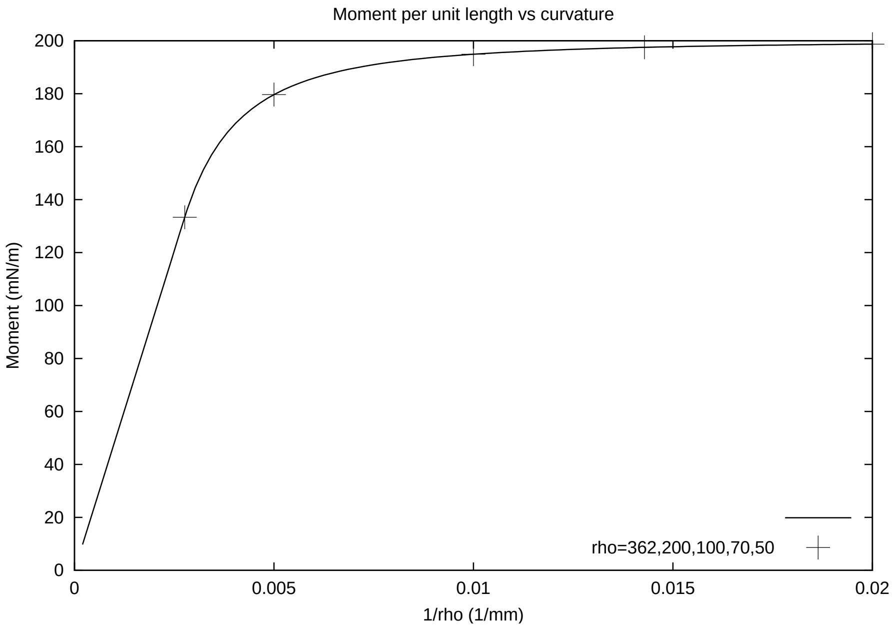

FIG. 5 – Moment as a function of curvature for $\sigma_y = 200$ MPa, $E = 72400$ MPa on a sheet of thickness 2 mm (crosses indicate respectively radii of 362, 200, 100, 70, 50 mm)

Geometry and material parameters can be modified in the boxes below the figures and new simulations performed with the "GO" button (about 15-20 secs waiting). Results are the moment-curvature relation on the left and the stress field at maximum and unloaded conditions on the right (x_2 label should be replaced by x_3). M_{max} and final radius of curvature (after unloading) R_{final} are also indicated.

1. Check the results obtained in Part I for a perfectly plastic material. Try various geometries and material properties. Consider the example of an aluminium alloy (given in Part I and of a construction steel ($\sigma_y = 400$ MPa, $E = 200000$ MPa). Try an higher strength material, like with $\sigma_y = 800$ MPa, $E = 200000$ MPa. Discuss the results in terms of the final radius obtained. Explain the reasons for differences with the analytical solution in Part I, question 6.
2. Introduce a linear isotropic hardening (slope b with $Q = 0$). Observe and discuss the corresponding modifications of stress repartition. Try different values for the hardening modulus.
3. Consider a non-linear isotropic hardening with for instance $\sigma_y = 400$ MPa, $Q = 400$ MPa, $b = 500$, $E = 200000$ MPa. Note the results. Check that unloading from point A to point B is linear. Observe and discuss the change in the stress repartitions in the thickness (print the sheet). Explain the stress values obtained at the surface in terms of material parameters.
4. Replace the isotropic hardening model of question 3 by the non-linear kinematic hardening model that gives exactly the same monotonic tensile curve. Indicate the assigned values for σ_y, C and D. Print the results and discuss the differences observed with question 3. What does arise during unloading ?
5. Perform the same kind of comparisons between isotropic and kinematic hardening for different values of σ_y (300, 350, 450, 500, 550), but enforcing the same limit value of 800 MPa for the maximum allowable stress in the material. Observe in which the two solutions practically coincide.
6. From the various observations made, and using the hardening law (in the particular case of isotropic hardening), define a possible way to obtain the local solution at any position x_3 in the thickness. A local expression involving $\varepsilon_p(x_3)$ as the only unknown, i.e. the uniaxial plastic strain

ε_{p11} at each position (recall that the accumulated plastic strain p in the hardening law is the time integral of $\dot{\varepsilon}_{p11}$ under such conditions, so that under a monotonic plastic loading we have $p = \varepsilon_p$). This local expression must distinguish between the elastic zone $|x_3| < a$ and the plastic zone $|x_3| > a$.

During plastic flow, from question 4 in Part I, we have $\varepsilon(x_3) = \frac{\sigma_y}{E}\frac{x_3}{a}$ in the whole thickness. Value of a that plays role in the above expression is unknown. An additional closure expression can be associated with it, given by the corresponding bending moment.

Assuming that we control the calculation with a (decreasing from $a = h$ to $a = 0$) explain how the non linear expression obtained can be solved at every point x_3, then the resultant bending moment and curvature obtained. Making a program for that in any language (Excel or Matlab or other) should be a "must", but is not a demand of the project.